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Hidden Matter Condensation
Effects on Supersymmetry Breaking

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ABSTRACT

We investigate the effects of hidden matter condensation on supersymmetry breaking in supergravity models derived from free fermionic strings. We find that the minimum of the effective potential in the modulus direction depends strongly on only one parameter which is fixed by the hidden sector. For nonpositive values of the parameter the potential is unstable which constrains realistic models severely. For positive and decreasing values which correspond to more and/or lighter hidden matter, T_R increases whereas T_I is periodic and depends on the parameter very weakly. Supersymmetry can be broken in the matter direction with a stable vacuum only if the fields which give mass to the hidden matter are light and have modulus independent Kahler potentials. Then, for a wide range of model parameters, supersymmetry is mainly broken by hidden matter condensation in the matter direction rather than by hidden gaugino condensation in the modulus direction.

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1. Introduction

There is growing though circumstantial experimental evidence for believing that supersymmetry (SUSY) is a true symmetry of Nature. If SUSY exisits, it must be broken which can only happen nonperturbatively[1] due to the well-known nonrenormalization theorems[2]. The best candidate for dynamical SUSY breaking seems to be condensation effects in the hidden sectors of supergravity (SUGRA)[3] or superstring[4] models.

SUSY breaking by gaugino condensation[5] in the hidden sectors of SUGRA or string derived SUGRA models have been extensively examined in the literature [6-12]. In this scenario, when the hidden gauge group becomes strong at a hierarchically small scale Λ_H compared with the Planck scale M_P, gaugino condensates which break SUSY form. The effects of SUSY breaking in the hidden sector are communicated to the observable sector by gravity and possibly by nonrenormalizable terms in the superpotential which are proportional to inverse powers of M_P. The effective nonperturbative superpotential can be obtained from the symmetries of the underlying gauge theory, i.e. by satisfying the anomalous and nonanomalous Ward identities[7,8]. On the other hand, one can find the effective scalar potential by simply substituting the VEV of the gaugino condensate into the SUGRA Lagrangian[6]. A very important ingredient in this scenario is target space duality which is a symmetry of the string to all orders in perturbation theory and also assumed to hold nonperturbatively[13]. As a result, the nonperturbative effective superpotential has to be invariant under target space duality, a property which restricts its possible moduli dependence severely. The dilaton dependence of the nonperturbative superpotential is determined by the running coupling constant which is a function of the dilaton[14].

For a hidden sector without matter, hidden sector gaugino condensation gives an effective scalar potential which has minima or vacua at $T_R \sim 1.23$ and its dual value $T_R \sim 0.81$. On the other hand, the scalar potential is not stable in the dilaton direction resulting in $S_R \to \infty$ [10]. The dilaton potential can be stabilized either

by adding a dilaton independent term to the superpotential [10] or by having more than one hidden gauge group[9]. These vacua break SUSY in the modulus but not the dilaton direction, i.e. $\langle F_T \rangle \neq 0$ but generically $\langle F_S \rangle = 0$. In addition, in all versions of this scenario, the cosmological constant is nonvanishing (and negative).

In string or string derived SUGRA models the generic situation is a hidden sector with matter and not the pure gauge case. The effect of hidden matter on SUSY breaking must be taken into account unless all hidden matter multiplets are heavy and decouple at the condensation scale. This case has been considered assuming that SUSY is not broken in the matter direction in Refs. [8,15]. There, it was argued that the presence of hidden matter does not change the results of the pure gauge case significantly. In this paper, we will show that this is not so at least in SUGRA derived from free fermionic superstrings[16] . The presence of hidden matter with nonzero mass (as required to have a stable vacuum) has two effects. First, it modifies the nonperturbative superpotential of the pure gauge case in a well–known way[8]. Second, as was shown in Refs. [17,18] it may result in SUSY breaking in the matter direction if the fields which give mass to hidden matter do not decouple at the condensation scale Λ_H.

In Ref. [18] the effects of hidden matter condensation on SUSY breaking were investigated in a generic SUGRA model derived from free fermionic superstrings. The F–terms in the overall modulus and matter directions were obtained from the effective superpotential and compared with each other without finding the vacuum (or the scalar, dilaton and moduli VEVs) by minimizing the effective scalar potential. It was shown that SUSY can mainly be broken by hidden matter condensation in the matter direction rather than by hidden gaugino condensation in the modulus direction. Whether the former or the latter is dominant depends on the parameters of the string or SUGRA model such as the hidden gauge group, hidden matter content and their masses and the vacuum, i.e. the VEVs for the dilaton, moduli and scalar fields which are fixed dynamically. In this paper, we extend our previous work by examining the effect of hidden matter condensation on SUSY breaking with or without matter F–terms again in a generic SUGRA

model derived from free fermionic strings. We make no attempt to solve the dilaton stability or the cosmological constant problems. We assume that the dilaton VEV is stabilized at $S \sim 1/2$ by some mechanism which may be either of the two mentioned above. We perform our analysis by obtaining the effective scalar potential for the different cases we examine and minimizing it numerically.

We find that when SUSY is not broken in the matter direction, the location of the minima, maxima and saddle points in the modulus direction depend mainly on the parameter $d' = (6N - 2M - t)/4\pi N$. Here N, M and t are the hidden gauge group (assumed to be $SU(N)$), the number of hidden matter multiplets in the fundamental representation and the power of $\eta(T)$ in the determinant of the hidden matter mass matrix respectively. In this case, T_R at the maxima and saddle points increase with decreasing d' which corresponds to more and /or lighter hidden matter. T_R at the minima behave the same way except that in addition there appear new solutions with small (i.e. < 0.2) T_R for small values (i.e. $< 1/7$) of d'. For all critical points T_I is periodic and (almost) independent of d'.

When SUSY is broken in the matter and modulus directions, i.e. there is a nonvanishing matter F–term F_{ϕ_i}, the results depend on the Kahler potential of the matter field ϕ_i. If $K(\phi_i, \phi_i^\dagger)$ depends on the modulus, there is no stable minimum in the T_R direction. On the other hand, if the Kahler potential of ϕ_i is canonical there are stable vacua for all positive values of d'. The stability of the vacuum requires that if SUSY is broken in the matter direction, the same matter fields must have canonical Kahler potentials. In this case, T_R, T_I at the minima behave similarly to the $F_{\phi_i} = 0$ case but now there are no new minima at small T_R for small d'. This is a crucial difference because it is exactly at these points that $F_T > F_{\phi_i}$ whereas for all minima with large T_R, $F_{\phi_i} > F_T$ for most of the model parameter space. Thus, we conclude that when $F_{\phi_i} \neq 0$, the dominant SUSY breaking mechanism is hidden matter condensation in the matter direction rather than hidden gaugino condensation in the modulus direction. We also find that, whether matter F–terms vanish or not, for $d' \leq 0$, there is no minimum in the T_R direction , i.e. $T_R \to \infty$ which gives $F_T = 0$ (and $F_{\phi_i} = 0$). Requiring a stable vacuum in the modulus

direction constrains the hidden sectors of possible realistic models severely.

The paper is organized as follows. In Section 2, we give the features which are common to all realistic SUGRA models derived from free fermionic strings. These include the matter and moduli content, the superpotential, the Kahler potential and the supersymmetric vacuum around the Planck scale. In Section 3, we review SUSY breaking by hidden gaugino condensation in the pure gauge case. The effects of hidden matter condensation without a matter F–term are considered in Section 4. We minimize the effective scalar potential numerically and find that the presence of hidden matter modifies the pure gauge case results significantly. In Section 5, we examine hidden matter condensation effects in the presence of a nonvanishing F–term in the matter direction. We consider two cases: matter with a modulus dependent Kahler potential and with a canonical one. We find the conditions under which the matter or modulus F–term dominates SUSY breaking. Section 6 contains a discussion of our results and our conclusions.

2. Supergravity models derived from free fermionic strings

The low–energy effective field theory limit of superstrings is given by $N=1$ SUGRA models with a gauge group and field content fixed by the underlying string. The string derived SUGRA model is defined in addition by three functions: the Kahler potential K, the superpotential W and the gauge function f[3]. SUGRA derived from free fermionic strings have some generic features which we outline below. These can also be considered as assumptions about the string models which we examine in this paper. We consider a SUGRA model derived from a free fermionic superstring [16] with the following properties:

a) The spectrum of the SUGRA model which is given by the massless spectrum of the superstring is divided into three sectors. The first one is the observable sector which contains states with charges under the Standard Model gauge group. The second one, the hidden sector, contains singlets of the Standard Model group which are multiplets of the hidden gauge group. These two sectors are connected only by

nonrenormalizable terms in the superpotential, gravity and gauged $U(1)$s which are broken around the the Planck scale. Therefore, once the gauged $U(1)$s are broken the two sectors are connected only by interactions proportional to inverse powers of M_P. The third sector generically contains a large number of matter fields (ϕ_i) which are Standard Model and hidden gauge group singlets. ϕ_i are connected to the observable and hidden states only through gauge $U(1)$s (in addition to gravity and nonrenormalizable interactions which are proportional to inverse powers of M_P). Therefore, ϕ_i behave effectively as hidden matter once the $U(1)$s are broken. Throughout the paper we call the fields ϕ_i matter (not observable or hidden). It is the F–terms of these ϕ_i that we are interested in when we examine SUSY breaking in the matter direction.

b) The hidden sector contains one (or more) $SU(N)$ (or other nonabelian gauge) group(s) with M copies of matter $(h_i, \bar{h}_i)$ in the vector representations $N + \bar{N}$. In the following, we consider only the one gauge group case since in realistic models part of the hidden gauge group must be broken by VEVs which are essential for obtaining CKM mixing[19]. In any case our results are not changed by the introduction of additional hidden gauge groups with matter. The case of multiple hidden gauge groups has been extensively examined in Ref. (15). The net effect of additional hidden gauge groups is to stabilize the dilaton potential which we assume in the following. The hidden matter states obtain masses from nonrenormalizable terms, W_n, of the type given in Eq. (2) below. This is essential since a supersymmetric gauge theory with massless matter does not have a well–defined vacuum[20]. As a result of the nonrenormalizable terms, the hidden matter mass matrix is nonsingular and the model has a stable vacuum. In addition, $M < 3N$ so that the hidden gauge group is asymptotically free and condenses at the scale $\Lambda_H \sim M_P exp(8\pi^2/bg^2)$ where $b = M - 3N$.

c) Realistic free fermionic strings generically have a number of untwisted moduli in their massless spectrum. These show up in the low–energy SUGRA model as fields which do not appear in the superpotential to any order in perturbation theory. One moduli always present in all string models is the dilaton. The exact type and

number of the other untwisted moduli depend on the boundary conditions for the internal fermions and are model dependent [21]. In realistic free fermionic string models one can have up to three T type and three U type moduli, one pair for each compactified torus (sector). In the following we will assume to have only one untwisted modulus which is the overall modulus T for simplicity. Thus, we will deal with target space duality under only the overall modulus T. It is straightforward to generalize this case to the one with any number of untwisted moduli of either T or U type. On the other hand, there are free fermionic strings for which some or all tori (sectors) do not have any moduli. Matter fields arising from these sectors have canonical Kahler potentials which do not depend on moduli.

d) The superpotential is given by

$$W = W_{3,obs} + W_{3,hid} + W_n + W_{np}, \tag{1}$$

where the cubic superpotential W_3 is divided into two parts: one which contains only the observable states and the other only the hidden states. W_n gives the nonrenormalizable terms ($n > 3$) in the superpotential and W_{np} gives the nonperturbative contributions due to gaugino and matter condensation in the hidden sector. Due to the supersymmetric nonrenormalization theorems[2], the only correction to the string tree level superpotential is W_{np}. Note that there are no renormalizable interactions between observable and hidden matter arising from the superpotential. We assume that the same is true also for ϕ_i and the Standard Model states. We also take the gauge function $f_{\alpha\beta} = S\delta_{\alpha\beta}$ at the string tree level[14]. Neglecting the string one loop corrections to $f_{\alpha\beta}$ do not change our results qualitatively.

e) The nonrenormalizable (order $n > 3$) terms in the superpotential are generically of the form

$$W_n = c_n g h_i \bar{h}_j \phi_{j_1} \phi_{j_2} \ldots \phi_{j_{n-2}} \eta(T)^{2n-6} M_v^{3-n}, \tag{2}$$

and are obtained from the world–sheet correlators

$$A_n \sim \langle V_1^f V_2^f V_3^b \ldots V_n^b \rangle, \tag{3}$$

which satisfy all the selection rules due to the local and global charges and Ising model operators as given by Ref. (22). In Eq. (2), c_n are calculable numerical coefficients of $O(1)$ and $\eta(T)=e^{-\pi T/12}\prod_n(1-e^{-2\pi nT})$ is the Dedekind eta function. In free fermionic strings, modular weights of matter fields under the overall modulus T, are given by the sum $\sum_{i=1}^3 Q_{\ell_i}$ where Q_{ℓ_i} give their R charges[23]. A generic feature of free fermionic strings is that all matter fields ϕ_i and h_i have modular weights -1. Thus, the cubic superpotential W_3 is automatically target space modular invariant. Nonrenormalizable terms W_n are rendered modular invariant by multiplying them by the required powers of $\eta(T)$ which has a modular weight of 1/2. In Eq. (2) the powers of $\eta(T)$ and M_v ($\sim M_P$ to be defined later) are such that the term W_n has modular weight -3 and dimension 3 as dictated by dimensional analysis and target space modular invariance. Note that these terms contain both observable and at least a pair of hidden sector states. Once the fields ϕ_i get VEVs (in order to have a supersymmetric vacuum at M_P as a result of the anomalous D–term as we will see below), they give masses to the hidden states $h_i,\bar{h}_i$. Consequently, all the $n>3$ terms of the type given by Eq. (2) can be seen as hidden matter mass terms. (In general, there can also be terms of the form $c_n\phi_{i_1}\phi_{i_2}\ldots\phi_{i_n}$, i.e. nonrenormalizable terms with only observable fields. These vanish in standard–like models [17] and we assume that they are not present in the following. Elimination of these terms is closely related to discrete symmetries which protect light quark masses[24]. If they do exist, they may destabilize the SUSY vacuum and break SUSY at very large scales which is phenomenologically a disaster.)

f) The Kahler potential (at tree level) is given by (for $\phi_i << T$)

$$K(S,S^\dagger,T,T^\dagger,\phi_i,\phi_i^\dagger)=-log(S+S^\dagger)-3log(T+T^\dagger)-\sum_i(T+T^\dagger)^{n_i}\phi_i\phi_i^\dagger, \quad (4)$$

where S,T and ϕ_i are the dilaton, (overall) modulus and matter fields respectively and n_i is the modular weight (under T) of the matter field ϕ_i. There are also models in which some sectors do not have any moduli. Matter fields coming from these

sectors have canonical Kahler potentials which do not depend on the modulus. The presence of such matter fields will be crucial for stablizing the scalar potential with matter F–terms in Section 5. The modulus and matter fields in Eq. (4) are in the "supergravity basis" and are related to the massless string states by well–known transformations [25,26].

g) The string vacuum is supersymmetric at the Planck scale, M_P and at the level of the cubic superpotential. This is guaranteed by satisfying the F and D constraints obtained from the cubic superpotential W_3 (which is trilinear in ϕ_i and h_i) and the local charges of the states. As we saw above, all nonrenormalizable terms in the superpotential, W_n, contain hidden matter bilinears. As a result, W_3 does not get any higher order corrections as long as the hidden gauge group does not condense at $\Lambda_H << M_P$ and W_3 is the exact superpotential until hidden sector condensation. The set of F and D constraints is given by the following set of equations [17]:

$$D_A = \sum_i Q_i^A |\langle \phi_i \rangle|^2 = \frac{-g^2}{192\pi^2} Tr(Q_A) \frac{1}{2\alpha'}, \tag{5a}$$

$$D^j = \sum_i Q_i^j |\langle \phi_i \rangle|^2 = 0, \tag{5b}$$

$$\langle W_3 \rangle = \langle \frac{\partial W_3}{\partial \phi_i} \rangle = 0, \tag{5c}$$

where ϕ_i are the matter fields and Q_i^j are their local charges. α' is the string tension given by $(2\alpha')^{-1} = g^2 M_P^2/32\pi = g^2 M_v^2$ and $Tr(Q_A) \sim 100$ generically in realistic string models. Eq. (5a) is the D constraint for the anomalous $U(1)_A$ which is another generic feature of free fermionic string models [27]. Note that the anomalous D–term arises at the string one loop level and therefore contains a factor of $g^2 = 1/4(S + S^\dagger)$. We see that some Standard Model singlet scalars must get Planck scale VEVs of $O(M_v/10)$ in order to satisfy Eq. (5a) and preserve SUSY around the Planck scale. Then, due to the other F and D constraints most of the other SM singlet scalars also obtain VEVs of $O(M_v/10)$. In this

manner, all gauge $U(1)$s which connect ϕ_i and $h_i, \bar{h}_i$ to the Standard Model states are broken spontaneously at the high scale $O(M_v/10)$. In addition, the scalar VEVs break target space duality spontaneously since they carry modular weights. These corrections to W_3 when they become nonzero, (i.e. when hidden matter condensates $\Pi_{ij} = h_i \bar{h}_j$ form) modify the cubic level F constraints in Eq. (5a) and may destabilize the original SUSY vacuum in the matter direction as was shown in Ref. (17).

3. Hidden sector gaugino condensation

The leading candidate for SUSY breaking in string derived SUGRA is hidden sector gaugino condensation[5]. In this section, we review the simplest possibility which is gaugino condensation in a hidden sector with a pure gauge group i.e. no hidden matter. As mentioned previously, realistic string models generically contain hidden matter in vectorlike representations. This more complicated case will be discussed in the following sections. Our purpose in reviewing the pure gauge case is to introduce the basic concepts and our notation.

In this scenario, due to the running of the coupling constant, the hidden gauge group condenses around the scale $\Lambda_H \sim M_P Exp(8\pi^2/bg^2)$, resulting in a gaugino condensate. The nonperturbative effective superpotential for the gaugino condensate Y^3 can be obtained from the symmetries (Ward identities) of the underlying gauge theory to be[7,8]

$$W_{np} = \frac{1}{32\pi^2} Y^3 log\{exp(32\pi^2 S)[c\eta(T)]^{6N} Y^{3N}\}, \tag{6}$$

where c is a constant. W_{np} has modular weight -3 as required since Y^3 and S have modular weights -3 and 0 respectively. All the fields which appear in W_{np} are scaled by M_v. The composite gaugino condensate superfield Y^3 can be integrated out by taking the flat limit $M_P \to \infty$ at which gravity decouples. In this limit,

SUGRA reduces to global SUSY whose vacuum is given by

$$\frac{\partial W_{tot}}{\partial Y} = 0. \tag{7}$$

The solution to the above equation gives the gaugino condensate in terms of S and T

$$\frac{1}{32\pi^2} Y^3 = (32\pi^2 e)^{-1} [c\eta(T)]^{-6} exp(-32\pi^2 S/N), \tag{8}$$

resulting in the nonperturbative superpotential

$$W_{np}(S,T) = \Omega(S) h(T), \tag{9}$$

with

$$\Omega(S) = -N exp(-32\pi^2 S/N), \tag{10a}$$
$$h(T) = (32\pi^2 e)^{-1} [c\eta(T)]^{-6}. \tag{10b}$$

The effective scalar potential due to W_{np} is given by

$$V = |F_S|^2 G^{-1}_{SS^\dagger} + |F_T|^2 G^{-1}_{TT^\dagger} - 3e^K |W|^2, \tag{11}$$

where $G = K + log|W|^2$, $W = W_{np}$ ($W_3 = 0$ in vacuum from Eq. (5)) and the F–terms are

$$F_k = e^{K/2}(W_k + K_k W), \tag{12}$$

for $k = S, T$. Using the above formula we find (from now on we use the notation $S = S_R + iS_I$ and $T = T_R + iT_I$)

$$F_S = \frac{1}{(2S_R)^{1/2}(2T_R)^{3/2}} h(T) \left(\Omega_S - \frac{\Omega}{2S_R} \right), \tag{13}$$

and

$$F_T = \frac{1}{(2S_R)^{1/2}(2T_R)^{3/2}}\Omega(S)\left(h_T - \frac{3h}{2T_R}\right). \tag{14}$$

Substituting the above F–terms into Eq. (11) gives the scalar potential

$$V = \frac{1}{16S_R T_R^3|\eta(T)|^{12}}\left\{|2S_R\Omega_S - \Omega|^2 + 3|\Omega|^2\left(\frac{T_R^2}{\pi^2}|\hat{G}_2|^2 - 1\right)\right\}, \tag{15}$$

where $\hat{G}_2 = G_2 - \pi/T_R$ and G_2 is the second Eisenstein function given by

$$G_2(T) = \frac{\pi^2}{3} - 8\pi^2\sum_n \sigma_1(n)e^{-2\pi nT}. \tag{16}$$

$\sigma_1(n)$ is the sum of the divisors of n and $G_2(T)$ arises due to

$$\frac{\partial\eta(T)}{\partial T} = -\frac{\eta(T)}{4\pi}G_2(T). \tag{17}$$

$\hat{G}_2(T)$ is a regularized version of $G_2(T)$ which has modular weight -2 (in contrast $G_2(T)$ does not have a well–defined modular weight)[11]. The vacuum is obtained by minimizing the scalar potential with repect to S and T. If any of the F–terms given by Eqs. (13) and (14) are nonzero in the vacuum, SUSY is broken spontaneously. It is well–known that the condition for a minimum in the S direction is given by[10]

$$S_R\Omega_S - \Omega = 0. \tag{18}$$

Note that the minimum in the S direction does not depend on the modulus T. With $\Omega(S)$ given by Eq. (10a) one finds that there is no (finite) minimum or (stable) vacuum since the solution to Eq. (18) requires $S_R \to \infty$. This is the dilaton stability problem and we will not try to solve it in this paper. It has been noted that the dilaton potential can be stabilized with a realistic dilaton VEV, i.e. $S_R \sim 1/2$ either by adding a constant term to $\Omega(S)$[10] or by having more than one hidden gauge group[15]. We stress that the condition for the minimum in the S direction automatically insures $F_S = 0$ which we will assume to hold in the following.

The minimization in the modulus direction gives

$$\frac{\partial V}{\partial T} = \frac{3}{32\pi S_R T_R^3} \frac{1}{|\eta(T)|^{12}} \{\hat{G}_2[|2S_R\Omega_S - \Omega|^2 + 3|\Omega|^2 \left(\frac{T_R^2}{\pi^2}|\hat{G}_2|^2 - 1\right)] + \frac{T_R}{\pi}|\Omega|^2[2|\hat{G}_2|^2 + T_R\hat{G}_2^*\hat{G}_{2T} + T_R\hat{G}_{2T}^*\hat{G}_2]\} = 0. \tag{19}$$

The first term in the curly brackets vanishes due to $F_S = 0$. From Eq. (19) we see that the minimum in the modulus direction is independent of the dilaton S. In addition, it is also independent of the hidden gauge group or N. The critical points of the potential V in Eq. (15) have been investigated [10]. There are maxima at $(T_R, T_I) = (\sqrt{3}/2, 1/2 + n)$ and saddle points at $(T_R, T_I) = (1, n)$ which are given by the solutions to $\hat{G}_2(T) = 0$. (Here n is an integer.) Both at the maxima and saddle points $F_T = 0$ since $F_T \propto \hat{G}_2(T)$ as it is seen from Eq. (14). The minima are given by solutions to Eq. (19) which are not solutions of $\hat{G}_2(T) = 0$. They are at $(T_R, T_I) = (\sim 1.23, n)$ and its dual $(T_R, T_I) = (\sim 0.81, n)$ which give $F_T \neq 0$. The maxima and saddle points appear at the self–dual (or fixed) points of target space duality due to the fact that $\hat{G}_2$ transforms covariantly under target space duality (or has a well-defined modular weight) and modular functions always have zeros at these fixed points. The minima, on the other hand, are not at the fixed points of target space duality. Therefore, target space duality which is spontaneously broken by the vacuum manifests itself by the presence of two minima which are connected to each other by target space duality.

4. Hidden sector gaugino and matter condensation

As mentioned in Section 2, free fermionic strings generically have hidden sectors which contain matter $h_i, \bar{h}_i$ in the vectorlike representations of the hidden gauge group. In addition, there are generic observable matter fields ϕ_i which give masses to hidden matter. In this section, we repeat the steps of the previous one taking into account the effects of hidden matter condensation. We assume that SUSY is not broken in the matter direction, i.e. $F_{\phi_i} = 0$. The case where $F_{\phi_i} \neq 0$ will be examined in the next section.

In the presence of hidden matter, when the hidden gauge group condenses at Λ_H, matter condensates $\Pi_{ij}=h_i\bar{h}_j$ form in addition to gaugino condensates Y^3. The nonperturbative effective superpotential obtained from the Ward identities and modular invariance becomes[7,8]

$$W_{np}=\frac{1}{32\pi^2}Y^3 log\{exp(32\pi^2 S)[c'\eta(T)]^{6N-2M}Y^{3N-3M}det\Pi\}-trA\Pi, \qquad (20)$$

where c' is a (new) constant and A is the hidden matter mass matrix given by the $n>3$ terms in Eq. (2). W_{np} has modular weight -3 as required since A and Π have modular weights -1 and -2 respectively. The last term corresponds to the sum of all the $n>3$ terms in Eq. (2). The observable matter fields ϕ_i which give masses to hidden matter appear only in the mass matrix A. In the flat limit $M_P\to\infty$, gravity decouples and one gets a globally SUSY vacuum at which (in addition to Eqs. (5a-c))

$$\frac{\partial W_{tot}}{\partial Y}=\frac{\partial W_{tot}}{\partial \Pi}=0, \qquad (21)$$

where $W_{tot}=W_3+W_{np}$. We can replace W_{tot} in Eq. (21) by W_{np} since W_3 does not contain Y^3 or Π. The $n>3$ terms, W_n which are the hidden matter mass terms, are already included in W_{np} through $trA\Pi$. The solutions to Eq. (21) are used to obtain the composite fields Y^3 and Π in terms of S,T and A

$$\frac{1}{32\pi^2}Y^3=(32\pi^2 e)^{M/N-1}[c\eta(T)]^{2M/N-6}[detA]^{1/N}exp(-32\pi^2S/N), \qquad (22)$$

and

$$\Pi_{ij}=\frac{1}{32\pi^2}Y^3A^{-1}_{ij}. \qquad (23)$$

Eqs. (22) and (23) are used to eliminate the composite fields in W_{np}

$$W_{np}(S,T)=\Omega(S)h(T)[detA]^{1/N}, \qquad (24)$$

where

$$\Omega(S)=-Nexp(-32\pi^2S/N), \qquad (25a)$$

$$h(T) = (32\pi^2 e)^{M/N-1}[c\eta(T)]^{2M/N-6}. \qquad (25b)$$

$detA$ is a product of mass terms given generically by Eq. (2). Thus, without any loss of generality, we can assume that it has the form

$$detA = kS_R^{-r}\phi_i^{s_i}\eta(T)^t \qquad r,s,t>0, \qquad (26)$$

where the S dependence is obtained from the relation $g^2 = 1/4S_R$ (at the string tree level and for level one Kac–Moody algebras). The parameters r and t can be expressed in terms of the more fundamental ones, hidden sector parameters N, M and the order of nonrenormalizable mass terms n for a given model. ϕ_i denotes any matter field which appears in $detA$ and s_i is its power. k is a constant of $O(1)$ which is given by the product of the relevant c_n in Eq. (2). In fact, this is the form of $detA$ which was obtained from the explicit model of Ref. (17) with $r=7$, $t=22$ and $s_i = 1,5$ depending on the field ϕ_i. (In general, $detA$ is a sum of terms like that in Eq. (26).) We see that there is a new S and T dependence in W_{np} due to $detA$. The new S dependence does not change the results of the previous section qualitatively. The scalar potential still has a minimum only at $S \to \infty$ which needs to be stabilized and $F_S = 0$ due to the minimization condition. On the other hand, the new T dependence leads to qualitative and quantitative changes as we will see below.

Now, there are two possibilities: either ϕ_i are heavier than Λ_H, i.e. $m_{\phi_i} >> \Lambda_H$ and they decouple at Λ_H or they are lighter than Λ_H, i.e. $m_{\phi_i} < \Lambda_H$ and they remain in the spectrum. In this section, we assume the former which has two consequences. First, since ϕ_i decouple at the condensation scale one can substitute their VEVs everywhere and forget about them. Second, there is no SUSY breaking in the ϕ_i direction, i.e. $F_{\phi_i} = 0$. The second case in which ϕ_i are light will be examined in the next section. Then, ϕ_i do not decouple and become dynamical fields like S and T. In both cases we assume that the hidden matter states $h_i, \bar{h}_i$ do not decouple from the spectrum at Λ_H since otherwise obviously there can only be gaugino condensation.

In W_{np} all the information about the matter condensates, Π_{ij}, and the observable fields ϕ_i is contained in the term $detA$. When $m_{\phi_i} >> \Lambda_H$ and ϕ_i decouple, one simply substitutes the VEVs $\langle\phi_i\rangle$ obtained from the solution to the F and D constraints in $detA$. ϕ_i are longer dynamical fields since at the scale Λ_H these heavy fields cannot be excited but simply sit at their VEVs. In this sense, ϕ_i are similar to the composite fields Y^3 and Π which are also eliminated from W_{np}. All ϕ_i do is to give masses to the hidden matter states $h_i, \bar{h}_i$ through their VEVs. As a result, in this case the only effect of matter condensates Π_{ij} is to change the scale of the gaugino condensate Y^3 through $detA$.

Using Eq. (12) we obtain for the dilaton F–term

$$F_S = \frac{e^{-\phi_i \phi_i^\dagger/4T_R}}{(2S_R)^{1/2}(2T_R)^{3/2}} h(T)[detA]^{1/N}\{\Omega_S - \frac{\Omega}{2S_R} + \Omega(log[detA]^{1/N})_S\}. \quad (27)$$

The first two terms in the curly brackets are the usual ones coming from gaugino condensation. The last term gives the contribution of the matter condensates (through $detA$) to F_S. Assuming the above form for $detA$ we get

$$\frac{\partial(log[detA]^{1/N})}{\partial S} = -\frac{r}{NS_R}. \quad (28)$$

It is easy to see that this additional term can be absorbed into a redefinition of b and does not change the F_S or the dilaton potential qualitatively. Once again the dilaton has a runaway potential with $S_R \to \infty$ which should be stabilized by some mechanism.

For the F–term in the modulus direction we find

$$F_T = \frac{e^{-\phi_i \phi_i^\dagger/4T_R}}{(2S_R)^{1/2}(2T_R)^{3/2}} \Omega(S)[detA]^{1/N}\{h_T - \frac{3h}{2T_R} + \frac{\phi_i \phi_i^\dagger}{4T_R^2} h + h(log[detA]^{1/N})_T\}. \quad (29)$$

As for F_S, the first two terms in the curly brackets arise from gaugino condensation whereas the last two come from matter Kahler potential $K(\phi_i, \phi_i^\dagger)$ and hidden

matter condensation respectively. From Eq. (26) for $detA$ we obtain for the last term

$$\frac{\partial(log[detA]^{1/N})}{\partial T} = -\frac{t}{4\pi N} G_2(T). \tag{30}$$

Combined with the power of $\eta(T)$ in $h(T)$ in Eq. (25b), $2M/N - 6$, these two terms modify the behavior in the modulus direction. The F–term in the modulus direction is now given by

$$F_T = \frac{e^{-\phi_i \phi_i^\dagger/4T_R}}{(2S_R)^{1/2}(2T_R)^{3/2}} \Omega(S)[detA]^{1/N} h(T) d' \left(G_2(T) - \frac{3}{2T_R d'} + \frac{\phi_i \phi_i^\dagger}{4T_R^2 d'} \right), \tag{31}$$

where $d' = (6N - 2M - t)/4\pi N$ which gives the scalar potential

$$V = \frac{e^{-\phi_i \phi_i^\dagger/2T_R}}{16 S_R T_R^3 |\eta(T)|^{8\pi d'}} |[detA]^{1/N}|^2 \{ |2S_R \Omega_S - \Omega - \frac{2\Omega r}{N}|^2 \\ + |\Omega|^2 \left(\frac{4d'^2 T_R^3}{(3T_R - \phi_i \phi_i^\dagger)} |G_2(T) - \frac{3}{2T_R d'} + \frac{\phi_i \phi_i^\dagger}{4T_R^2 d'}|^2 - 3 \right) \}. \tag{32}$$

We see that the effect of hidden matter condensates and their mass terms is simply to change the function $\hat{G}_2(T)$ to $G_2(T) - 3/2T_R d' + \phi_i \phi_i^\dagger / 4T_R^2 d'$ where d' is fixed by the hidden gauge group (N), the matter content of the hidden sector (M) and the hidden mass terms (t) in Eq. (26). The matter VEVs $\langle \phi_i \rangle$ are fixed by the F and D–terms of the of the superpotential to be $\sim M/10$ at the perturbative level. The additional nonperturbative scalar potential is much smaller than the perturbative one and therefore cannot change the matter VEVs by much. Any supersymmetric string vacuum contains a large number (of $O(10)$) of VEVs and therefore for our calculations we take $\langle \phi_i \phi_i^\dagger \rangle \sim 0.2$. We have numerically checked that our results are not sensitive to the exact value and number of the VEVs as long as they are nonzero and in a realistic range.

Note that, as expected, for $M = t = 0$ and $\langle \phi_i \rangle = 0$, $G_2'(T) \to \hat{G}_2(T)$ and the potential in Eq. (32) reduces to the pure gauge result given by Eq. (15).

In complete analogy with the pure gauge case, now the behavior in the modulus direction is determined by the function $G_2(T) - 3/2T_R d' + \phi_i \phi_i^\dagger / 4T_R^2 d'$. Minimizing in the dilaton direction, we find that

$$\frac{\partial V}{\partial S} \propto \Omega_S - \frac{\Omega}{2S_R} - \Omega(log[detA]^{1/N})_S, \tag{33}$$

which means that $F_S = 0$ in vacuum as in the pure gauge case.

The minimization condition in the T direction now reads (defining $G'(T) = G_2(T) - 3/2T_R d' + \phi_i \phi_i^\dagger / 4T_R^2 d'$)

$$\frac{\partial V}{\partial T} = \frac{e^{-\phi_i \phi_i^\dagger / 2T_R}}{16 S_R T_R^3} \frac{|[detA]^{1/N}|^2}{|\eta(T)|^{8\pi d'}} \{ d' G_2' [|2S_R \Omega_S - \Omega - \frac{2\Omega r}{N}|^2 + |\Omega|^2 \left(\frac{4d'^2 T_R^2}{3} |G_2'|^2 - 3 \right)]$$
$$+ \frac{2d' T_R}{3} |\Omega|^2 [2|G_2'|^2 + T_R G_2'^* G_{2T}' + T_R G_{2T}'^* G_2'] \} = 0, \tag{34}$$

compared to Eq. (19). Writing Eq. (34) we made the approximation $3T_R \sim 3T_R - \phi_i \phi_i^\dagger$ for simplicity. The numerical analysis was performed for the exact potential without this simplification. The first term in the curly brackets vanishes because $F_S = 0$ in the presence of hidden matter.

The maxima and the saddle points of V which were given by the solution to

$$\hat{G}_2(T) = G_2(T) - \frac{\pi}{T_R} = 0, \tag{35}$$

in the pure gauge case are now given by the solution to

$$G'(T) = G_2(T) - \frac{3}{2T_R d'} + \frac{\phi_i \phi_i^\dagger}{4T_R^2 d'} = 0. \tag{36}$$

Note that $G'(T)$ does not have a well–defined modular weight (i.e. not a modular function like $\hat{G}_2(T)$) due to the VEVs of ϕ_i in the matter Kahler potential $K(\phi_i, \phi_i^\dagger)$ and $detA$ which break target space duality spontaneously. As a result, these points are no longer the fixed points of target space duality but simply solutions of Eq.

(36). Also, we see that the location of the maxima and saddle points depend mainly on the parameter d' and not on the string model parameters N, M and t separately. In addition, there is a weak dependence on the scalar VEVs $\langle \phi_i \phi_i^\dagger \rangle$ (fixed to be ~ 0.2) which we have numerically found to be not important.

We find that as d' decreases, i.e. the hidden matter content (M) increases and/or the hidden masses decrease (t increases), T_R at the maxima and saddle points increase. The maxima of the pure gauge case at $(1, n)$ are now at (T_{Rmax}, n) where T_{Rmax} is given in Table 1 for some values of d'. The saddle points which for the pure gauge case were at $(\sqrt{3}/2, 1/2+n)$ are now given by $(T_{Rsp}, 1/2+n)$ where T_{Rsp} are given for the same values of d' in Table 1. It is easy to see from Table 1 that as there is more hidden matter (increasing M) and/or the hidden masses become smaller (larger t) the maxima and saddle points appear at larger values of T_R. Note that the value of T_I depends on d' very weakly since the minimization condition for T_I (and not T given by Eq. (34)) is almost d' independent because at the minimum $d'T_R$ is (almost) constant. We also see that the T_I values are periodic with a period of 1 since the modular functions which appear in the effective scalar potential, $\eta(T)$ and $G_2(T)$, are periodic functions of T_I with the same period.

The minima of the pure gauge case which were at $(\sim 1.23, n)$ and $(\sim 0.81, n)$ are now given by the solutions to Eq. (36) which are not zeros of $G'(T)$. Again the T_R at the minimum mainly depends on d' rather than on N, M and t separately. A numerical study of the scalar potential gives the values in Table 1 for the T_{Rmin} for some values of d'. We see that T_{Rmin} increases with decreasing d' but at small (i.e. $< 1/7$) values of d' a new minimum with very small (i.e. < 0.2) T_R appears in addition to the one with large T_R. Once again, as for the maxima and saddle points, T_I at the minima are periodic and (almost) independent of d'. Now however, contrary to the pure gauge case, the minima for a given d' are not connected to others by target space duality transformations. This is because target space duality is spontaneously broken by the VEVs of ϕ_i which results in a scalar potential which does not have a well–defined modular weight. Note also that the minimum for the case with no hidden matter, i.e. $M = t = 0$, in Table 1 does

not reproduce the pure gauge result because of the Kahler potential term for the scalars ϕ_i in the potential.

When $d' \leq 0$, we find that there is no minimum in the modulus direction, i.e. $T_R \to \infty$. The reason is the change of sign in the exponential of T_R which comes from $\eta(T)$. In this case one also gets $F_T = 0$ so there is no SUSY breaking in the modulus direction. Unless T_R is stabilized, one does not have a well-defined vacuum and cannot obtain SUSY breaking. This is a new stability problem in addition to the one for the dilaton. Compared to the dilaton case, this result is much more difficult to modify since the modulus dependence of the nonpertirbative superpotential is strongly constrained due to target space duality. In order to avoid this situation, we require $d' > 0$ or $6N - 2M - t > 0$ which severely constrains the hidden sectors of possible realistic models. Without hidden matter masses, i.e. $t = 0$, $d' > 0$ always since this is required for the asymptotic freedom of the hidden sector. For M copies of hidden matter in the fundamental representation $N + \bar{N}$, using Eq. (26) for $detA$ and Eq. (2) for the matter mass terms, we have $t = \sum_{i=1}^{M}(2n_i - 6)$ where n_i is the order at which the mass term appears in the superpotential. Requiring that the hidden matter remains in the spectrum around the condensation scale Λ_H, we get $n \sim 7 - 8$ or larger thus giving $t \sim 10M$. The condition for stability in the T_R direction loosely becomes $3N - 5M > 0$ which is a rather strong condition on the hidden sector of realistic models. Of course, M is the number of light hidden matter multiplets and not their overall number. A given string model cannot be ruled out on the basis of the massless string spectrum using the above condition since some or all of the hidden matter can get large masses and decouple due to matter VEVs. Once all hidden masses are found though, the above condition must be satisfied in order to get realistic SUSY breaking and a stable vacuum in the modulus direction.

We see that the minimum of the effective scalar potential is not at a fixed point of target space duality neither for the pure gauge case (due to $K(\phi_i, \phi_i^\dagger)$) nor for the case with hidden matter (due to $detA$). This is a result of the spontaneous breaking of target space duality by the scalar VEVs. On the other hand, it is

well–known that free fermionic strings correspond to orbifold models formulated at the fixed points of target space duality i.e. $T_R = 1$ (in units of M_v). How should one interpret the above results obtained from the low–energy effective field theory? First note that the overall modulus T in the supergravity basis that appears in the low–energy scalar potential is related to the overall modulus t_s in the string basis by[11,25,26]

$$t_s = \frac{T_c - T}{\bar{T}_c + T}, \tag{37}$$

where $T_c = 1$. Thus at the fixed point of target space duality $T = 1$ the VEV of the modulus in the string basis vanishes, $t_s = 0$. This is fine since modulus field t_s is the coefficient of the exactly marginal operator

$$: y^i \omega^i :: \bar{y}^i \bar{\omega}^i : \quad i = 1, \ldots, 6 \tag{38}$$

which deforms the original two dimensional free fermionic string action. Here $y^i, \omega^i, \bar{y}^i, \bar{\omega}^i$ are the internal world–sheet fermions which describe the compactified six dimensional manifold in the fermionic language[21,25]. For $T = 1$, $t_s = 0$ and one has a free fermionic string as expected. Once $T \neq 1$, we get a nonzero t_s and therefore the free fermionic string is deformed by the above Abelian Thirring interaction. Consequently, one should understand the result of this section as follows. Hidden gaugino and matter condensation and the scalar VEVs produce an effective scalar potential which perturbs the initial value of the modulus to the values of T_{Rmin} given in Table 1 for different values of the parameter d'. As a result, the free fermionic string is perturbed by the above Abelian Thirring interaction with the coefficient given by the value of the modulus in the string basis t_s. The low–energy model with the scalar VEVs, hidden masses and hidden matter and gaugino condensates is not described by the original free fermionic string but by one which is perturbed by the corresponding exactly marginal operator.

5. SUSY breaking by hidden matter condensation

In this section we consider the second case mentioned above in which $m_{\phi_i} < \Lambda_H$ and ϕ_i remain in the spectrum. Then, ϕ_i should be treated as dynamical fields similar to S and T since they can be excited due to their small masses. Now $W = W(S, T, \phi_i)$ where from Eq. (24) all the ϕ_i dependence is in the term $detA$ which arises due to the matter condensates Π_{ij}. In this case, in addition to F_S and F_T, one should also evaluate F_{ϕ_i} since it can be nonzero in the vacuum resulting in SUSY breaking in the matter direction. It may also be possible to break SUSY mainly by hidden matter condensation in the matter direction rather than by hidden gaugino condensation in the modulus direction, i.e. $F_{\phi_i} > F_T$ in vacuum. We consider two cases depending on the Kahler potential of the matter fields ϕ_i whose F–terms are nonzero. First, we investigate the effective potential when the matter Kahler potential depends on the modulus as given by Eq. (4). Then, we repeat the same analysis for matter with canonical Kahler potential. Both cases and a mixture of the two are possible depending the details of the string model.

The hidden matter condensates, through the term $detA$, induce an F–term in the matter direction, ϕ_i

$$F_{\phi_i} = \frac{e^{-\phi_i \phi_i^\dagger / 4T_R}}{(2S_R)^{1/2}(2T_R)^{3/2}} [\Omega(S) h(T) [detA]^{1/N} \\ \times \left(\frac{s_i}{N\phi_i} + \frac{\phi_i^\dagger}{4T_R} \right) + (W_{3\phi_i} + K_{\phi_i} W_3)]. \qquad (39)$$

This is the result obtained in Ref. (17) where the effect of matter condensation on F_{ϕ_i} due to hidden matter mass terms was examined. The last two terms simply give the contribution coming from the cubic superpotential which vanishes for the solution to the F and D constraints in Eqs. (5a-c) before the hidden gauge group condenses. Generically the F and D flat solutions give $\langle \phi_i \rangle \sim M_v/10$, a scale which is set by the coefficient of the anomalous D–term in Eq. (5a). The nonperturbative scalar potential also contains the scalars ϕ_i but since it is much smaller than the tree level potential we assume that it does not modify the VEVs of ϕ_i appreciably. Therefore, we set the second paranthesis above to zero. We

see that for realistic values of s_i and N (i.e. of the same order of magnitude) the first term in the second paranthesis in Eq. (39) (which corresponds to the W_k piece in F_k) dominates the second one when $\langle\phi_i\rangle \sim M_v/10$ unless T_R is very small (i.e. < 0.01). In order for these two terms to cancel each other, one needs either $\langle\phi_i\rangle \sim M_v$ and $T_R \sim 1$ or $\langle\phi_i\rangle \sim M_v/10$ and $T_R \sim 0.01$ and a considerable amount of fine tuning. F_{ϕ_i} obviously arises solely from matter condensation since its origin is the hidden matter mass term $trA\Pi$ in Eq. (20).

In Ref. (17), it was shown that F_{ϕ_i} may be nonzero in vacuum once W_n or hidden matter mass terms are taken into account. The reason is that, the $n > 3$ terms give corrections to the cubic superpotential, W_3, which modify the cubic level F constraints. For large orders n these corrections turn the F constraints into an inconsistent set of equations. As a result, the new set of F constraints up to a given order $n > 3$, cannot be solved simultaneously for any set of scalar VEVs. In particular, at the minimum, there is always a nonzero F_{ϕ_i} for some ϕ_i and SUSY is spontaneously broken in the matter direction. The amount of SUSY breaking in the matter direction given by F_{ϕ_i} depends on the parameters of the model such as M, N, t and s_i.

We stress that ϕ_i are connected to the squarks and sleptons either through gravity or the broken gauge $U(1)$s. (We neglect the nonrenormalizable interactions which do not affect our results.) Since the $U(1)$s are broken at the high scale of $O(M_v/10)$, interactions of ϕ_i with the squarks and sleptons are suppressed by $O(10/M_v)$ and thus are almost as weak as gravity. Therefore, this mechanism has all the characteristics of hidden sector supersymmetry breaking rather than visible sector supersymmetry breaking[28].

Now, for $F_{\phi_i} \neq 0$ (and $F_S = 0$), when the Kahler potential of ϕ_i is given by Eq. (4), the scalar potential becomes

$$V = \frac{e^{-\phi_i \phi_i^\dagger/2T_R}}{16 S_R T_R^3 |\eta(T)|^{8\pi d'}} |[detA]^{1/N}|^2 |\Omega|^2 \{-2T_R|\frac{s_i}{N\phi_i} + \frac{\phi_i^\dagger}{4T_R}|^2$$

$$+\left(\frac{4d'^2T_R^3}{(3T_R-\phi_i\phi_i^\dagger)}|\{G_2(T)-\frac{3}{2T_Rd'}+\frac{\phi_i\phi_i^\dagger}{4T_R^2d'}|^2-3\right)\}. \tag{40}$$

The minimum in the modulus direction is given by

$$\begin{aligned}\frac{\partial V}{\partial T} =&\frac{e^{-\phi_i\phi_i^\dagger/2T_R}}{32\pi S_R T_R^3}\frac{|[detA]^{1/N}|^2|\Omega|^2}{|\eta(T)|^{8\pi d'}}\{d'G_2'[|\frac{s_i}{N\phi_i}+\frac{\phi_i^\dagger}{4T_R}|^2+\left(\frac{4d'^2T_R^2}{3}|G_2'|^2-3\right)]\\ &-\frac{|\phi_i|^2}{8T_R^2}-|\frac{s_i}{N\phi_i}+\frac{\phi_i^\dagger}{4T_R}|^2+\frac{2d'T_R}{3}|\Omega|^2[2|G_2'|^2\\ &+T_RG_2'^*G_{2T}'+T_RG_{2T}'^*G_2']\}=0,\end{aligned} \tag{41}$$

where we used $3T_R \sim 3T_R-\phi_i\phi_i^\dagger$ in the above expression for simplicity. We stress that the numerical minimization was performed without this simplification.

Compared to the previous case, the nonzero F_{ϕ_i} modifies the effective potential by the first term in the curly brackets in Eq. (40). The behavior in the modulus direction is altered significantly due to the modulus dependence of the matter Kahler potential. A numerical analysis shows that the above potential does not have a minimum in the T_R direction (at least for realistic values of the model parameters). Thus, there is a stability problem in the modulus direction if $F_{\phi_i} \neq 0$ and the matter Kahler potential depends on the modulus. This stability problem arisies because of the first term in the curly brackets in Eq. (40), i.e. the modulus dependent matter Kahler potential. For realistic values of F_{ϕ_i} (which are given by $3 < s_i/N\phi_i < 30$) and other model parameters, this term destroys the minima we found for the $F_{\phi_i}=0$ case in the previous section. This is easy to understand since the new term is given (for $T_R > 0.5$ which is true for all minima) by $-T_R$ times a large number of $O(10)$ which is enough to eliminate the minima. In light of the above result, since we want a potential which is stable in the T_R direction, we will assume that $F_{\phi_i}=0$ for all matter fields ϕ_i with a modulus dependent Kahler potential.

It seems that there cannot be a nonzero matter F–term which results in a stable potential in the T_R direction. This is not so as we will now show. As

we saw above, the source of the problem is the modulus dependent matter Kahler potential. Therefore, the simplest solution is to find matter with a Kahler potential which does not depend on the modulus. In fact, there are matter fields (which we denote ϕ_i again for simplicity) with canonical Kahler potentials i.e.

$$K(\phi_i, \phi_i^\dagger) = -\sum_i \phi_i \phi_i^\dagger. \tag{42}$$

These are untwisted matter fields which arise from sectors with all their moduli projected out due to the twists of the basis vectors which define the free fermionic string model[21]. For example, if there are four complex world–sheet fermions, there is one sector with no moduli and all matter fields arising from this sector have canonical Kahler potentials. If there are six complex world–sheet fermions all untwisted moduli are projected out. Then, all untwisted matter fields have canonical Kahler potentials which is the optimal case.

In this optimal case, when there are no untwisted moduli, the matter F–term becomes

$$\begin{aligned} F_{\phi_i} = \frac{e^{-\phi_i \phi_i^\dagger}}{(2S_R)^{1/2}(2T_R)^{3/2}} &[\Omega(S) h(T) [detA]^{1/N} \\ &\times \left(\frac{s_i}{N\phi_i} + \frac{\phi_i^\dagger}{M_v^2} \right) + (W_{3\phi_i} + K_{\phi_i} W_3)]. \end{aligned} \tag{43}$$

The only difference between the above formula and Eq. (39) is in the Kahler term in front and the second term in the paranthesis which are independent of the modulus. The effective scalar potential becomes

$$\begin{aligned} V = &\frac{e^{-\phi_i \phi_i^\dagger}}{16 S_R T_R^3 |\eta(T)|^{8\pi d'}} |[detA]^{1/N}|^2 |\Omega|^2 \{ |\frac{s_i}{N\phi_i} + \frac{\phi_i^\dagger}{M_v^2}|^2 \\ &+ \left(\frac{4d'^2 T_R^2}{3} |G_2(T) - \frac{3}{2T_R d'}|^2 - 3 \right) \} \end{aligned} \tag{44}$$

We see that, due to the modulus independent Kahler potential, the problematic factor $(K_{\phi_i \phi_i^\dagger})^{-1} = -2T_R$ is absent in this case. Now the minimum of V depends

on the value of F_{ϕ_i} in addition to the parameter d'. From Eq. (39) we see that this is fixed by the combination $s_i/N\phi_i$. In Table 2, we give the minima of the scalar potential for some values of d' and three realistic values of F_{ϕ_i} given by $(s_i/N\phi_i)^2 = 10, 10^2, 10^3$. We find that the location of the minima does not depend strongly on $s_i/N\phi_i$ between these values. Since $K_{\phi_i\phi_i^\dagger}$ and F_{ϕ_i} do not depend on T (except for the Kahler potential term), the effect of a nonzero F_{ϕ_i} is to change the -3 term in the scalar potential to $(s_i/N\phi_i)^2-3$. From Table 2, we see that similarly to the $F_{\phi_i} = 0$ case, as d' decreases, i.e. as there is more hidden matter and/or hidden matter becomes lighter, T_R at the minima increase. Also as expected, T_I is periodic and depends on d' very weakly. The main difference which is crucial for our purposes is the absence of the minima with small values of T_R for small values of d'. Note that now there are no minima connected to the ones in the table by target space duality, i.e. $T \to 1/T$ because the potential in Eq. (44) does not have a well–defined modular weight. From Table 2, we also see that the value of T_R at the minima for a given d' increases slowly with the value of F_{ϕ_i}. In addition, as in the $F_{\phi_i} = 0$ case, for $d' \leq 0$ the scalar potential in Eq. (39) is unstable in the T_R direction. Therefore, we must require that the hidden sector satisfies $d' > 0$ or loosely $3N - 5M > 0$ as before.

One can also have a mixed case, i.e. some of the matter fields (ϕ_i) have Kahler potentials which depend on moduli while the rest (ψ_i) have canonical Kahler potentials. In this case, for reasons of stability in the T_R direction, we assume that $F_{\psi_i} \neq 0$ whereas $F_{\phi_i} = 0$. The scalar potential is now given by

$$V = \frac{e^{-\phi_i\phi_i^\dagger/2T_R}}{16S_RT_R^3|\eta(T)|^{8\pi d'}}|[detA]^{1/N}|^2|\Omega|^2\{-|\frac{s_i}{N\psi_i} + \frac{\psi_i^\dagger}{M_v^2}|^2 \\ + \left(\frac{4d'^2T_R^3}{(3T_R - \phi_i\phi_i^\dagger)}|\{G_2(T) - \frac{3}{2T_Rd'} + \frac{\phi_i\phi_i^\dagger}{4T_R^2d'}|^2 - 3\right)\}. \tag{45}$$

The minima of the potential in the modulus direction are given in Table 3 for three values of $s_i/N\psi_i$ as before. We find that the results are very similar to the previous case given in Table 2. Once again as d' decreases T_R at the minima

increase with no small T_R minima arising at small d'. T_I is almost independent of d' and periodic as before. As before, T_R at the minimum (for a given d') slightly increases with the value of the matter F–term. The two cases give qualitatively the same results with small quantitative differences which are not important. We conclude that the presence of matter with modulus dependent Kahler potentials does not affect our results as long as their F–terms vanish.

We find that, the scalar potential has stable minima when $F_{\phi_i} \neq 0$ if ϕ_i are matter fields whose Kahler potential is independent of the modulus. The presence of other matter with modulus dependent Kahler potentials does not alter the qualitative results as long as they do not have nonzero F–terms. For our purposes the most important numerical result is the absence of minima with $T_R < 0.2$ for the $F_{\phi_i} \neq 0$ case. This will play an important role when we examine the direction of SUSY breaking in field space.

In the presence of dynamical matter ϕ_i, one can ask several questions. First we see from Eq. (31) that there are two contrubutions to F_T; one from gaugino condensation and the other from matter condensation. What are the relative magnitudes of these? In particular, can the matter condensate contribution dominate that of the gaugino condensate in F_T? (We remind that F_{ϕ_i} arises solely from matter condensates.) A simple analysis shows that the gaugino condensation contribution is larger if $6N - 2M - t > 0$ and vice versa for all values of T_R at the minimum. Note that the two contributions enter F_T with opposite signs. We found above that unless this condition holds there is no stable minimum in the T_R direction and both F_T and F_{ϕ_i} vanish. Thus, in all cases with a stable minimum in the T_R direction, the hidden gaugino condensation contribution to F_T is larger than that of the hidden matter condensation. For small values of $6N - 2M - t$, when there is a large number of hidden matter multiplets and/or they are light, the two contributions are comparable. On the other hand, for large values of $6N - 2M - t$ the gaugino condensation contribution is dominant.

The second and more important question is for what range of model parameters

N, M, s_i, t etc. and vacuum (i.e. T_R, T_I) does one of the F–terms dominate the other, i.e. $F_T >> F_{\phi_i}$ or vice versa? The relative magnitude of these two F–terms gives the direction of SUSY breaking in field space (assuming as before that $F_S = 0$). If all matter ϕ_i have canonical Kahler potentials, from Eqs. (31) and (39) we find the ratio

$$\frac{F_T}{F_{\phi_i}} = \frac{N\phi_i}{s_i} d' \left(G_2(T) - \frac{3}{2T_R d'} \right), \tag{46}$$

whereas for the mixed case we have

$$\frac{F_T}{F_{\phi_i}} = \left(\frac{4N\phi_i T_R d'}{N|\phi_i|^2 + 4T_R s_i} \right) \left(G_2(T) - \frac{3}{2T_R d'} + \frac{\phi_i \phi_i^\dagger}{4T_R^2 d'} \right). \tag{47}$$

Using Eq. (46) for the case with only canonical Kahler potentials, we find that for most values of T_R at the minima in Table 2, $F_{\phi_i} > F_T$. For example, for the two limiting values of T_R in the table, i.e. $T_R \sim 0.8$ and $T_R \sim 4.7$ we obtain 0.2 and 0.04 for the ratio F_T/F_{ϕ_i} (assuming $N = s_i$ for simplicity). For smaller values of d' which gives larger T_R the ratio becomes even smaller. Of course, the ratio of F–terms also depends on the value of N/s_i which we took to be one. For example, at $T_R \sim 0.8$ if $N > 5s_i$ one gets $F_T > F_{\phi_i}$. This becomes more difficult when there is more and/or lighter hidden matter. Then, d' is small which gives large T_R and this requires a large ratio of N/s_i which is very difficult (if not impossible) to realize. For example, for $T_R > 1$ one needs $N > 10s_i$ or $N > 10$ in order to get $F_T \sim F_{\phi_i}$. On the other hand, the rank of the hidden gauge group is ≤ 11 which shows that this case is marginal whereas for larger T_R, $F_T < F_{\phi_i}$ always. It is only for small T_R that F_T can be naturally larger than F_{ϕ_i}. This is due to the very sharp increase (in absolute value) of $G_2(T)$ with decreasing T. These are exactly the minima which appear for small d' when $F_{\phi_i} = 0$ as we saw in the previous section. Now, however, when F_{ϕ_i} is nonvanishing, we find that these minima disappear due to the modification of the -3 term in the scalar potential by the $|s_i/N\phi_i|^2$ factor. As a result, for a wide range of parameters N, M, s_i, t and

$\phi_i \sim M_v/10$ we find that $F_{\phi_i} > F_T$. Repeating the above analysis for the mixed case using Eq. (47) and the results from Table 3 we get essentially identical results. This means that the ratio of the F–terms, F_T/F_{ϕ_i}, is not sensitive to the presence of matter with modulus dependent Kahler potential as long as these do not have nonzero F–terms.

The effect of the matter F–term is small only when $(s_i/N\phi_i) \sim 1$ and then the -3 factor is not changed by much. This situation is similar to the $F_{\phi_i} = 0$ case and there are minima with small T_R for small d'. From Eq. (46) we find that for $(s_i/N\phi_i) \sim 1$, F_T is the dominant SUSY breaking effect if the vacuum is given by the minimum with the small T_R and not the large one. Assuming $\phi_i \sim M_v/10$, this means that $N \sim 10 s_i$. For F_T to be dominant, we need at least an $SU(10)$ hidden gauge group and also that each scalar appear in the determinant of the hidden matter mass matrix only once. As we remarked in the previous paragraph, this is a marginal case at best which covers a small part of the parameter space.

6. Conclusions and discussion

In this paper, we investigated the effects of hidden matter condensation on SUSY breaking in SUGRA models derived from free fermionic strings. We found that the location of the critical points of the effective scalar potential depend mainly on one parameter, $d' = (6N - 2M - t)/4\pi$. Here N, M and t give the hidden gauge group, the number of hidden matter multiplets and the power of $\eta(T)$ in $det A$ where A is the hidden matter mass matrix respectively. The other parameter which is given by the VEVs of the scalar fields which give mass to hidden matter was taken to be $\phi_i \phi_i^\dagger \sim 0.2$ since there are in general a large number of scalar VEVs with $O(1/10)$ (in units of M_v) fixed by the anomalous D–term. We numerically checked that all our results depend very weakly on the value of $\phi_i \phi_i^\dagger$ as long as it is nonzero and in a realistic range.

When SUSY is not broken in the matter direction, we found that as d' decreases, i.e. when there is more and/or lighter hidden matter, T_R at the maxima

and saddle points of the effective scalar potential increase such that $d'T_R$ is about constant. Contrary to the case of a hidden sector with a pure gauge group, the minima and saddle points do not appear at the fixed points of target space duality since this is spontaneously broken by the scalar VEVs which give masses to hidden matter. T_R at the minima also increase with decreasing d' but for values smaller than $\sim 1/7$ new minima with small T_R appear. T_I at all the critical points are periodic since the modular functions which enter the scalar potential are so. In addition, T_I at these points depend very weakly on d'. When $d' \leq 0$, we found that there is no stable minimum in the T_R direction, i.e. $T_R \to \infty$. This stability problem is much more severe than the one for the dilaton since the modulus dependence of the effective nonperturbative superpotential is severely restricted by target space duality. In order to avoid this problem, realistic models must satisfy $d' > 0$ or $6N - 2M - t > 0$ which is a strong restriction on their hidden sectors. One cannot rule out string models on the basis their massless spectrum since part or all of the hidden matter can get large masses from scalar VEVs. Thus, the above condition should be used only for the part of the hidden sector which does not decouple from the theory at the condensation scale.

When SUSY is also broken in the matter direction, i.e. $F_{\phi_i} \neq 0$ in addition to $F_T \neq 0$, the results depend on the Kahler potential of the matter fields ϕ_i. If $K(\phi_i, \phi_i^\dagger)$ depends on moduli there is no stable minimum for the scalar potential in the T_R direction. This is true for all matter fields which arise from sectors with moduli. In order to get a stable potential, one must assume that all such matter fields have vanishing F–terms. This can happen if they do not enter the hidden mass matrix and therefore have vanishing F–terms due to the cubic level constraints. On the other hand, there are models in which some or all sectors are without moduli. In that case, the Kahler potential of matter fields coming from these sectors is canonical. We find that the scalar potential has a stable minimum in the presence of nonzero matter F–terms if they correspond to fields with canonical Kahler potentials. The presence of additional matter with modulus dependent Kahler potentials does not destroy the stability as long as their F–terms

as zero. The behavior of the critical points is very similar to the case with $F_{\phi_i} = 0$ as is seen from the Tables 1,2 and 3. We also find that the dependence of the minima on the value of F_{ϕ_i} is weak.

When $F_{\phi_i} \neq 0$, the most important numerical result for our purposes is the absence of minima with small (i.e. < 0.2) T_R. As a result of this, we find that for a wide range of model parameters $F_{\phi_i} > F_T$. Only for $N > 5s_i$ with very little hidden matter or for $N > 10s_i$, $F_T \geq F_{\phi_i}$. We conclude that for most of the parameter space SUSY is mainly broken by hidden matter condensation in the matter direction rather than by hidden gaugino condensation in the modulus direction.

We saw that in the presence of many hidden matter multiplets with small masses, the minima of the scalar potential are at large (> 1) T_R. Vacua with large T_R are desirable for obtaining large string threshold corrections[29] to the running coupling constants of the Standard Model gauge group. It is well–known that string unification occurs around 10^{17} GeV which is an order of magnitude larger than the scale predicted by the minimal supersymmetric extension of the Standard Model. This discrepancy can be eliminated without introducing extra states only by having large string threshold corrections which require large T_R. As we saw above, large values of T_R, which are very difficult to obtain without hidden matter, occur naturally when there is hidden matter which condenses. One can also turn this argument around and find the range of T_R required to get unification of coupling constants from string threshold corrections. This will give the realistic range of d' which in turn gives possible values of the string model parameters N, M, t etc. However, a given value of d' does not fix the parameters since there are different combinations of them which result in the same d'.

At the TeV scale, the only way to find out the direction of SUSY breaking in field space is to examine the sparticle masses (or soft–SUSY breaking parameters in general). It is well–known that these exhibit distinct patterns when SUSY is broken dominantly in the dilaton or the moduli directions[30]. For the former the

soft–SUSY breaking masses are all equal whereas for the latter they depend on the modular weights of the observable fields and in general are not equal to each other. One can extend these ideas to SUSY breaking in the matter direction and find the behavior of soft–SUSY breaking masses in this case. This would require information about the dependence of observable matter Kahler potentials on the fields ϕ_i. For observable matter coming from the twisted sectors the form of the Kahler potential has been conjectured to have a simple dependence on ϕ_i[31]. It is therefore plausible that SUSY breaking by hidden matter condensation in the matter direction leads to a pattern of sparticle masses which differs from the other two. In that case, one would be able to look for signs of this SUSY breaking mechanism around the TeV scale.

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